# Vv557 Methods of Applied Mathematics II

## Green Functions and Boundary Value Problems

## Assignment 5

**Exercise 5.1**

Calculate the Fourier transforms of the following elements in $\mathcal{S}'(\mathbb{R})$:

i) $\begin{cases} e^{-\varepsilon x} & x \geq 1, \\ 0 & x < 1, \end{cases} \qquad \varepsilon > 0,$

ii) $\sin(3x-2)$,

iii) $x^2 \cos(x)$,

iv) $xH(x-2)$,

v) $x^2\delta(x-1)$.

**Exercise 5.2**

A distribution $T \in \mathcal{D}'$ is said to be even if $T\varphi = T\widetilde{\varphi}$ for all test functions $\varphi$, where $\widetilde{\varphi}(x) = \varphi(-x)$. The distribution is said to be odd if $T\varphi = -T\widetilde{\varphi}$.

Show that if $T \in \mathcal{S}'$ is even (odd), then the Fourier transform $\widehat{T} \in \mathcal{S}'$ is even (odd).

**Exercise 5.3**

Consider the wave equation problem for a function $u \colon \mathbb{R}^2 \to \mathbb{R}$,

$$u_{tt} - u_{xx} = 0, \qquad u(x,0) = f(x), \qquad u_t(x,0) = g(x).$$

Take the Fourier transform of the equation with respect to the $x$-variable to obtain an ODE in the $t$-variable and solve the ODE to obtain

$$\widehat{u}(\xi,t) = \hat{f}(\xi)\cos(\xi t) + \frac{\hat{g}(\xi)}{\xi}\sin(\xi t).$$

Then calculate the inverse Fourier transform (in the distributional sense) to obtain a solution formula for $u(x,t)$.