Vv557 Methods of Applied Mathematics II

Green Functions and Boundary Value Problems

Assignment 4

Exercise 4.1

Calculate the Fourier transforms of the following elements in $L^1(\mathbb{R})$ (the theory of distributions is not needed):

i) $\Pi_{a,b}(x) = \begin{cases} 1 & a < x < b, \\ 0 & \text{otherwise}, \end{cases} \qquad a, b \in \mathbb{R}.$

ii) $e^{-a|x|}$, $a > 0$.

iii) e^{-ax^2}, $a > 0$.

iv) $\cos(x)e^{-x^2}$.

v) $\cos(2x)/(4 + x^2)$.

vi) the convolution of xe^{-x^2} and e^{-x^2}.

Exercise 4.2

Suppose that $(f * g)(x) = 0$ for all x, where $f, g \in \mathcal{S}(\mathbb{R})$. Does this imply that either $f \equiv 0$ or $g \equiv 0$? What if $f = g$?